IR Optimization

May 15th, 2013

But before we talk about IR optimization...

- Wrapping up IR generation

Three-Address Code

- Or "**TAC**"
- The IR that you will be using for the final programming project.
- High-level assembly where each operation has at most three operands.
- Uses explicit runtime stack for function calls.
- Uses vtables for dynamic dispatch.

Generating TAC

TAC Generation

- At this stage in compilation, we have
  - an AST,
  - annotated with scope information,
  - and annotated with type information.
- To generate TAC for the program, we do (yet another) recursive tree traversal!
  - Generate TAC for any subexpressions or substatements.
  - Using the result, generate TAC for the overall expression.

TAC Generation for Expressions

- Define a function **cgen**(*expr*) that generates TAC that computes an expression, stores it in a temporary variable, then hands back the name of that temporary.
- Define **cgen** directly for atomic expressions (constants, **`this`**, identifiers, etc.).
- Define **cgen** recursively for compound expressions (binary operators, function calls, etc.)

cgen for Basic Expressions

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```
cgen(k) = { // k is a constant
    Choose a new temporary t
    Emit( t = k );
    Return t
}
```

cgen for Basic Expressions

```
cgen(k) = { // k is a constant
    Choose a new temporary t
    Emit( t = k );
    Return t
}
cgen(id) = { // id is an identifier
    Choose a new temporary t
    Emit( t = id )
    Return t
}
```

cgen for Binary Operators

cgen for Binary Operators

cgen(e_1 + e_2) = {

Choose a new temporary t

Let t_1 = **cgen**(e_1)

Let t_2 = **cgen**(e_2)

Emit($t = t_1 + t_2$)

Return t

}

An Example

cgen(5 + x) = {

Choose a new temporary *t*

Let t_1 = **cgen**(5)

Let t_2 = **cgen**(x)

Emit ($t = t_1 + t_2$)

Return t

}

An Example

```
cgen(5 + x) = {
    Choose a new temporary t
    Let t1 = {
        Choose a new temporary t
        Emit( t = 5 )
        return t
    }
    Let t2 = cgen(x)
    Emit (t = t1 + t2)
    Return t
}
```

An Example

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cgen(5 + x) = {
    Choose a new temporary t
    Let t1 = {
        Choose a new temporary t
        Emit( t = 5 )
        return t
    }
    Let t2 = {
        Choose a new temporary t
        Emit( t = x )
        return t
    }
    Emit (t = t1 + t2)
    Return t
}
```

An Example

cgen(5 + x) = {

Choose a new temporary *t*

Let t_1 = {

Choose a new temporary *t*

Emit(*t* = 5)

return *t*

}

Let t_2 = {

Choose a new temporary *t*

Emit(*t* = x)

return *t*

}

Emit (*t* = t_1 + t_2)

Return t

}

```
_t0 = 5
_t1 = x
_t2 = _t0 + _t1
```

cgen for Statements

- We can extend the **cgen** function to operate over statements as well.
- Unlike **cgen** for expressions, **cgen** for statements does not return the name of a temporary holding a value.
  - *(Why?)*

cgen for Simple Statements

cgen for Simple Statements

cgen(*expr;*) = {
    cgen(*expr*)
}

`cgen` for **`while`** loops

cgen for **`while`** loops

cgen(**`while (`***expr***`)`** *stmt*) = {

}

cgen for **`while`** loops

cgen(**`while (`***expr***`)`** *stmt*) = {

Let L_{before} be a new label.

Let L_{after} be a new label.

}

cgen for **`while`** loops

cgen(**`while (`***expr***`)`** *stmt*) = {

Let L_{before} be a new label.

Let L_{after} be a new label.

Emit(L_{before} :)

Emit(L_{after} :)

}

cgen for **`while`** loops

cgen(**`while (`***expr***`)`** *stmt*) = {

Let L_{before} be a new label.

Let L_{after} be a new label.

Emit(L_{before} :)

Let *t* = **cgen**(*expr*)

Emit(**`IfZ`** *t* **`Goto`** L_{after})

Emit(L_{after} :)

}

cgen for **`while`** loops

cgen(**`while (`***expr***`)`** *stmt*) = {

Let L_{before} be a new label.

Let L_{after} be a new label.

Emit(L_{before} :)

Let *t* = **cgen**(*expr*)

Emit(**`IfZ`** *t* **`Goto`** L_{after})

cgen(*stmt*)

Emit(L_{after} :)

}

cgen for **`while`** loops

cgen(**`while (`***expr***`)`** *stmt*) = {

Let L_{before} be a new label.

Let L_{after} be a new label.

Emit(L_{before} **:**)

Let *t* = **cgen**(*expr*)

Emit(**`IfZ`** *t* **`Goto`** L_{after})

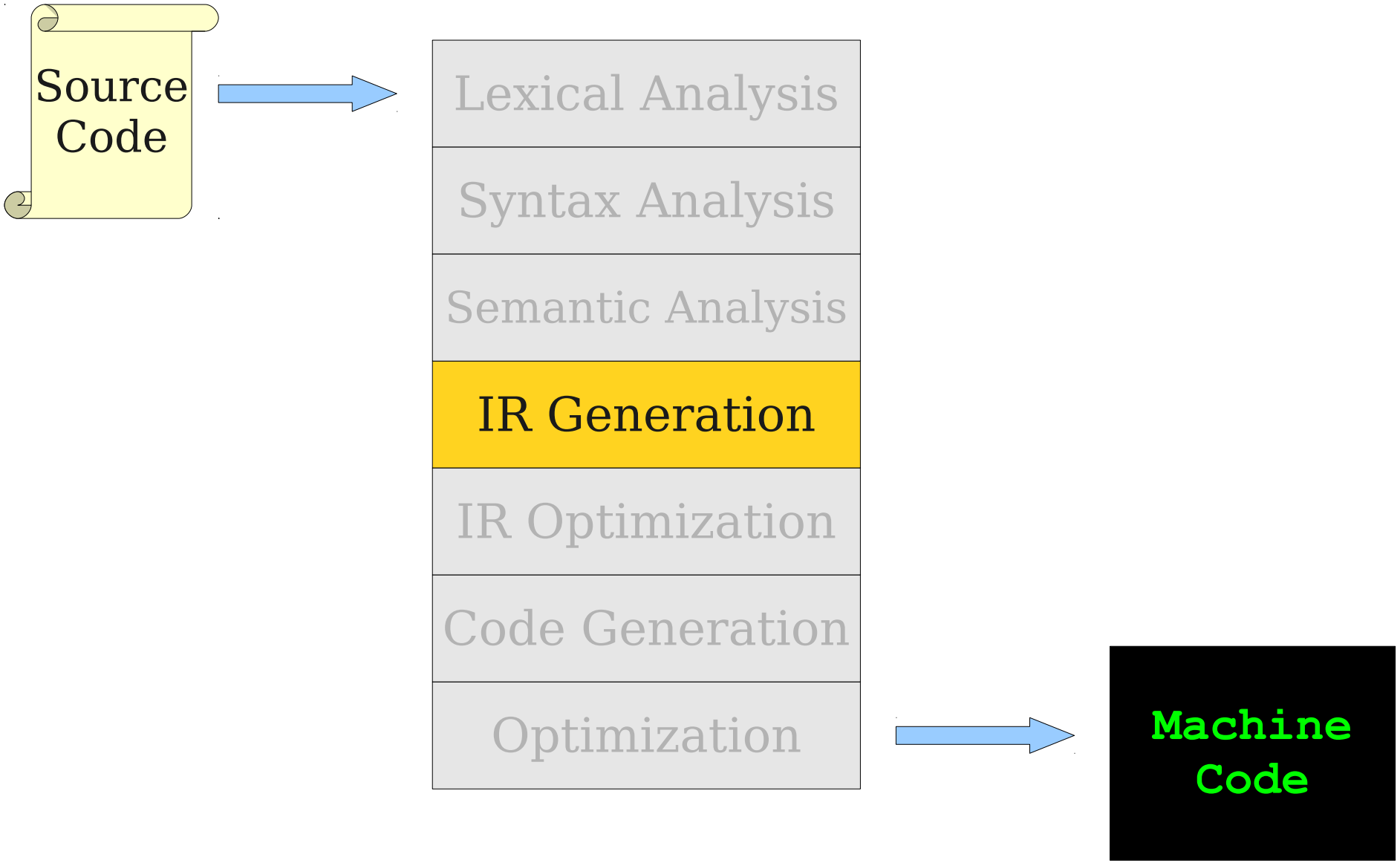
cgen(*stmt*)

Emit(**`Goto`** L_{before})

Emit(L_{after}**:**)

}

Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

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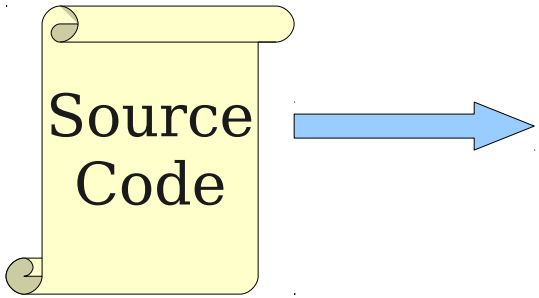

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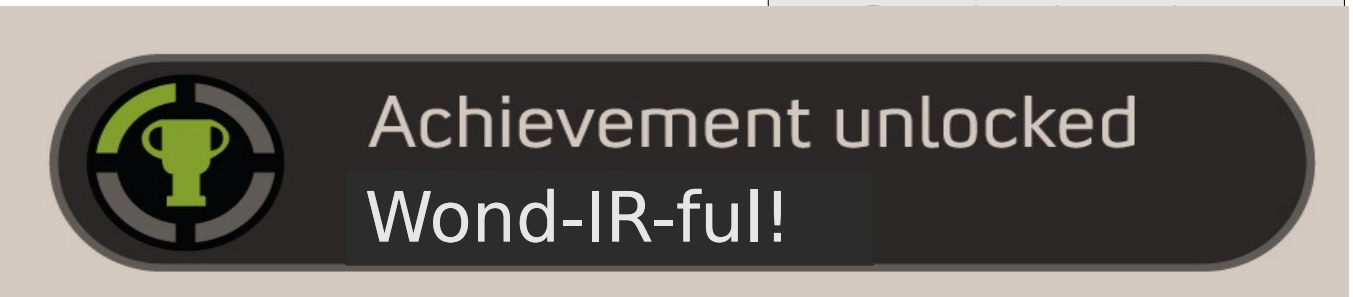

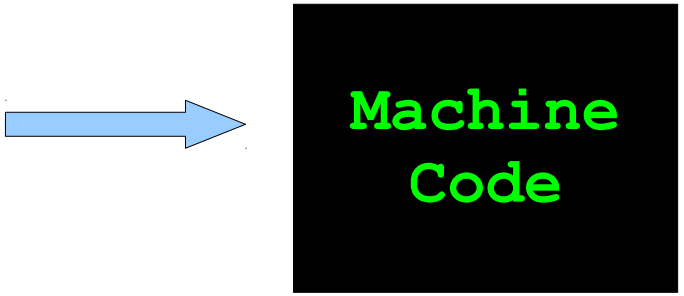

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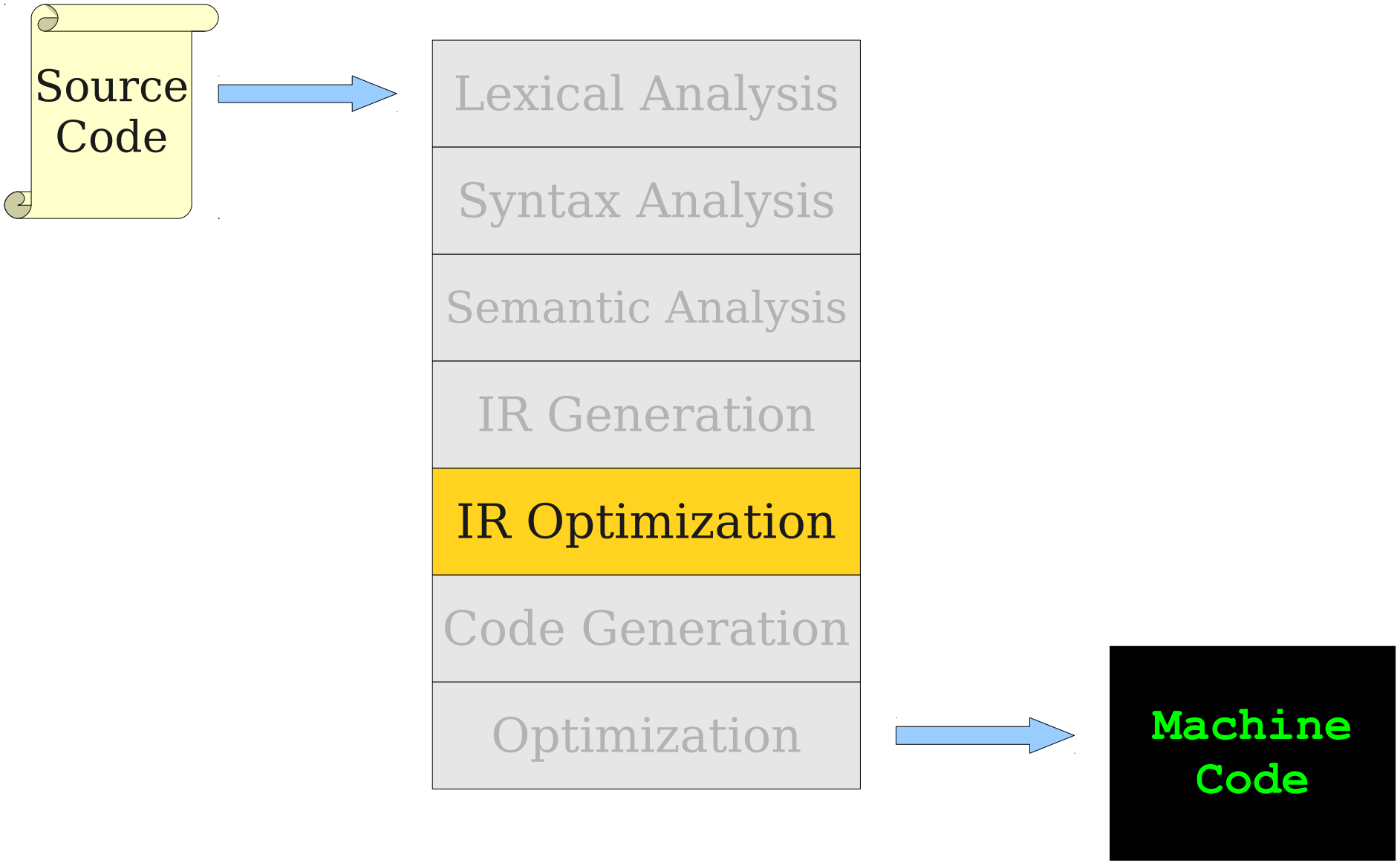
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IR Optimization

- **Goal**: Improve the IR generated by the previous step to take better advantage of resources.
- One of the most important and complex parts of any modern compiler.
- A very active area of research.

Sources of Optimization

- In order to optimize our IR, we need to understand why it can be improved in the first place.
- **Reason one:** IR generation introduces redundancy.
  - A naïve translation of high-level language features into IR often introduces subcomputations.
  - Those subcomputations can often be sped up, shared, or eliminated.
- **Reason two:** Programmers are lazy.
  - Code executed inside of a loop can often be factored out of the loop.
  - Language features with side effects often used for purposes other than those side effects.

Optimizations from IR Generation

```
int x;
int y;
bool b1;
bool b2;
bool b3;

b1 = x + x < y
b2 = x + x == y
b3 = x + x > y
```

Optimizations from IR Generation

```
int x;
int y;
bool b1;
bool b2;
bool b3;

b1 = x + x < y
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b3 = x + x > y
```

```
_t0 = x + x;
_t1 = y;
b1 = _t0 < _t1;

_t2 = x + x;
_t3 = y;
b2 = _t2 == _t3;

_t4 = x + x;
_t5 = y;
b3 = _t5 < _t4;
```

Optimizations from IR Generation

```
int x;
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b1 = x + x < y
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_t0 = x + x;
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b2 = _t2 == _t3;

_t4 = x + x;
_t5 = y;
b3 = _t5 < _t4;
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int x;
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bool b1;
bool b2;
bool b3;

b1 = x + x < y
b2 = x + x == y
b3 = x + x > y
```

```
_t0 = x + x;
_t1 = y;
b1 = _t0 < _t1;

b2 = _t0 == _t1;

b3 = _t0 < _t1;
```

```
while (x < y + z) {
    x = x - y;
}
```

Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
}
```

```
_L0:
    _t0 = y + z;
    _t1 = x < _t0;
    IfZ _t1 Goto _L1;
    x = x - y;
    Goto _L0;
_L1:
```

Optimizations from Lazy Coders

```
while (x < y + z) {
    x = x - y;
}
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```
_L0:
    _t0 = y + z;
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Optimizations from Lazy Coders

```
while (x < y + z) {
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}
```

```
    _t0 = y + z;
_L0:
    _t1 = x < _t0;
    IfZ _t1 Goto _L1;
    x = x - y;
    Goto _L0;
_L1:
```

The Challenge of Optimization

- A good optimizer
  - Should never change the observable behavior of a program.
  - Should produce IR that is as efficient as possible.
  - Should not take too long to process inputs.
- Unfortunately:
  - Even good optimizers sometimes introduce bugs into code.
  - Optimizers often miss “easy” optimizations due to limitations of their algorithms.
  - Almost all interesting optimizations are **NP**-hard or undecidable.

What are we Optimizing?

- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?

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- Optimizers can try to improve code usage with respect to many observable properties.
- What are some quantities we might want to optimize?
- **Runtime** (make the program as fast as possible at the expense of time and power)
- **Memory usage** (generate the smallest possible executable at the expense of time and power)
- **Power consumption** (choose simple instructions at the expense of speed and memory usage)
- Plus a lot more (minimize function calls, reduce use of floating-point hardware, etc.)

IR Optimization vs Code Optimization

- There is not always a clear distinction between what belongs to “IR optimization” versus “code optimization.”
- Typically:
  - IR optimizations try to perform simplifications that are valid across all machines.
  - Code optimizations try to improve performance based on the specifics of the machine.
- Some optimizations are somewhere in-between:
  - Replacing `x * 0.5` with `x / 2`

Optimization Overview

- Basic Concept
  - Control Flow Graphs
  - Basic blocks
- Local Optimization
- Global Optimization
- Advanced Topics
  - Data flow Analysis
  - Microarchitectural dependent optimizations
    - Register Allocation
    - Instruction selection
    - Instruction scheduling/pipelining
    - loop optimization

Semantics-Preserving Optimizations

- An optimization is **semantics-preserving** if it does not alter the semantics of the original program.
- Examples:
  - Eliminating unnecessary temporary variables.
  - Computing values that are known statically at compile-time instead of runtime.
  - Evaluating constant expressions outside of a loop instead of inside.
- Non-examples:
  - Replacing bubble sort with quicksort.
- The optimizations we will consider in this class are all semantics-preserving.

A Formalism for IR Optimization

- Every phase of the compiler uses some new abstraction:
  - Scanning uses regular expressions.
  - Parsing uses CFGs.
  - Semantic analysis uses proof systems and symbol tables.
  - IR generation uses ASTs.
- In optimization, we need a formalism that captures the structure of a program in a way amenable to optimization.

Visualizing IR

```
main:
    BeginFunc 40;
    _tmp0 = LCall _ReadInteger;
    a = _tmp0;
    _tmp1 = LCall _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    PushParam a;
    LCall _PrintInt;
    PopParams 4;
    EndFunc;
```

Visualizing IR

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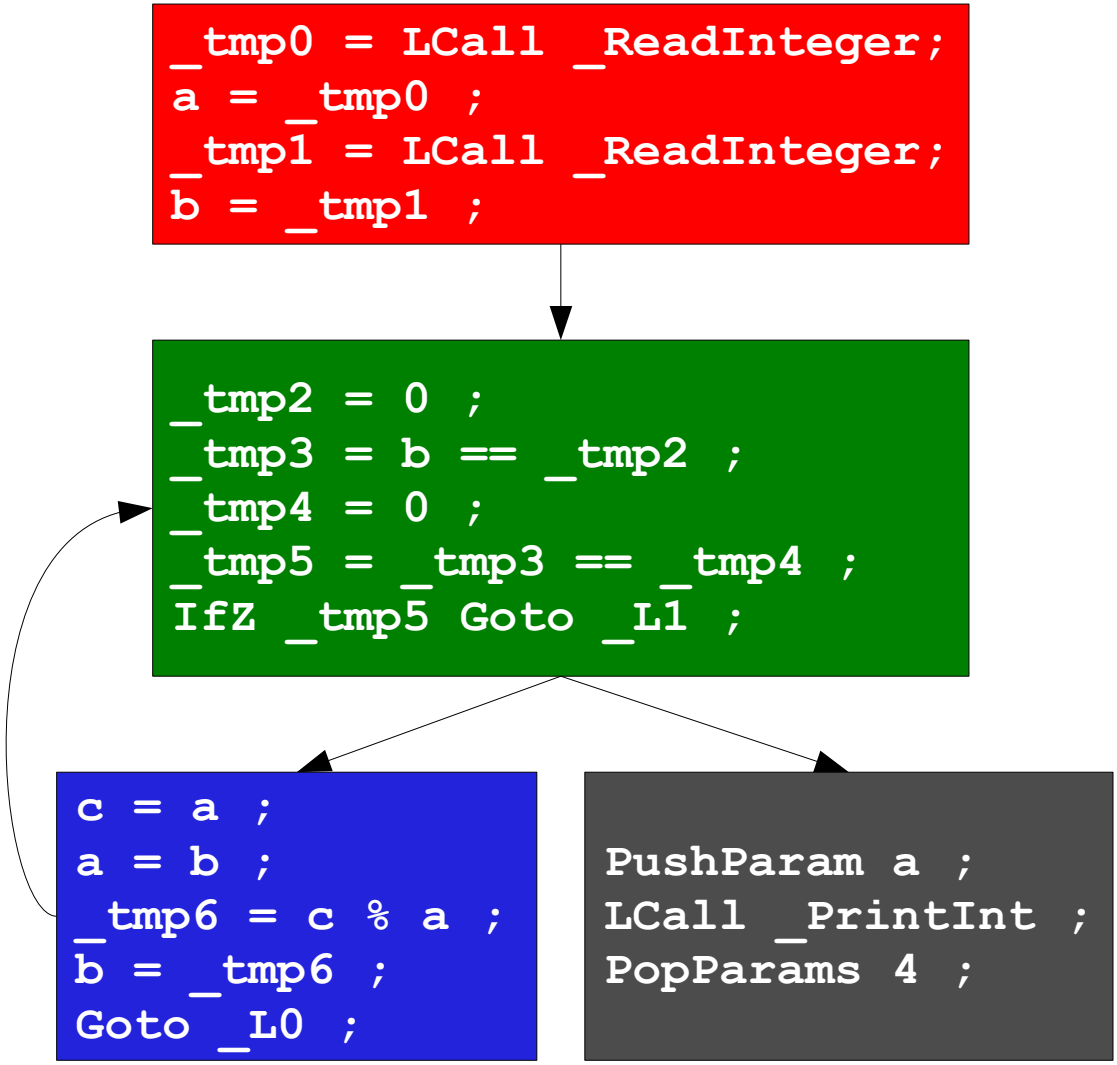

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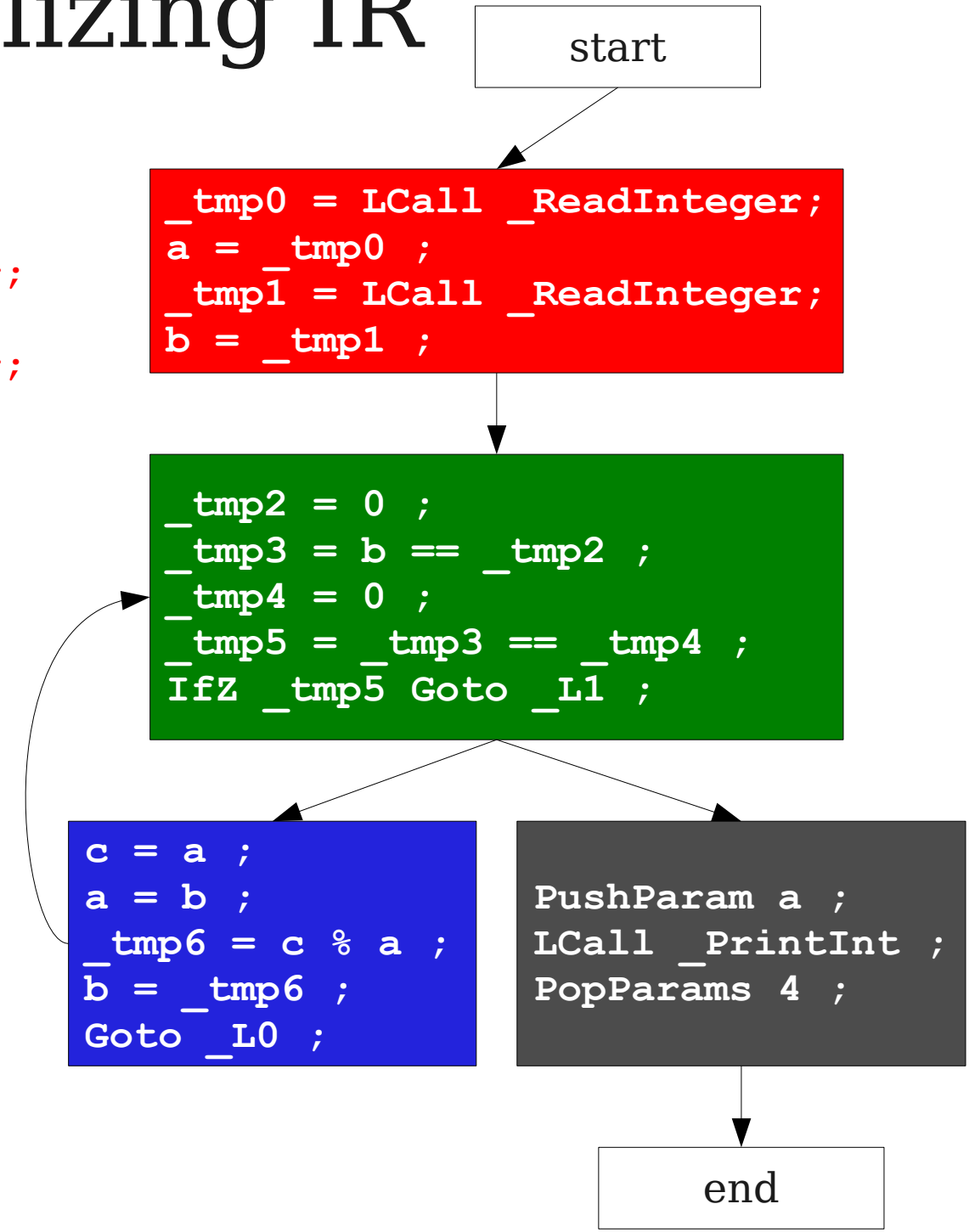


Basic Blocks

- A **basic block** is a sequence of IR instructions where
  - There is exactly one spot where control enters the sequence, which must be at the start of the sequence.
  - There is exactly one spot where control leaves the sequence, which must be at the end of the sequence.
- Informally, a sequence of instructions that always execute as a group.

Control-Flow Graphs

- A **control-flow graph** (CFG) is a graph of the basic blocks in a function.
  - The term CFG is overloaded – from here on out, we'll mean “control-flow graph” and not “context-free grammar.”
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block.
- There is a dedicated node for the start and end of a function.

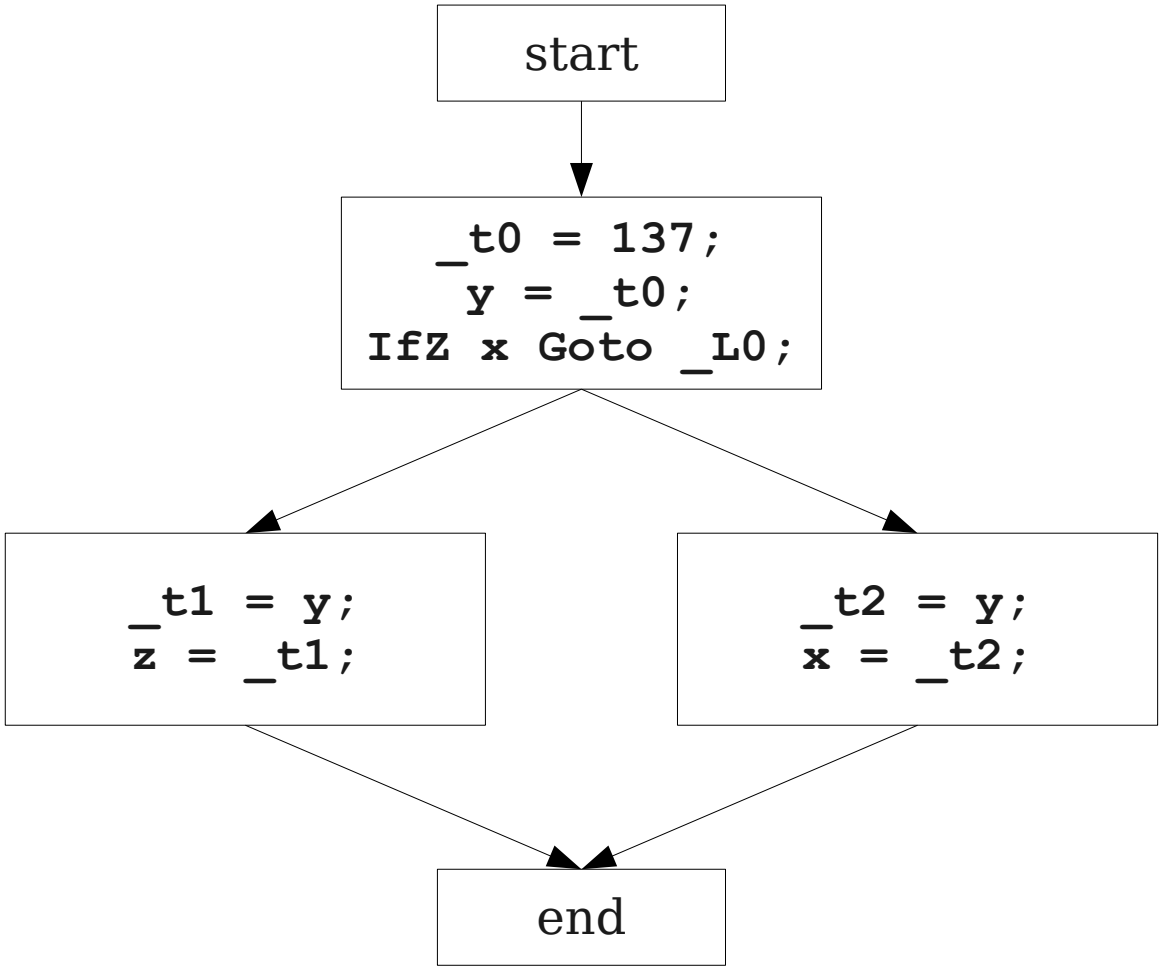
Types of Optimizations

- An optimization is **local** if it works on just a single basic block.
- An optimization is **global** if it works on an entire control-flow graph.
- An optimization is **interprocedural** if it works across the control-flow graphs of multiple functions.
  - We won't talk about this in this course.

Local Optimizations

```
int main() {
    int x;
    int y;
    int z;

    y = 137;
    if (x == 0)
        z = y;
    else
        x = y;
}
```



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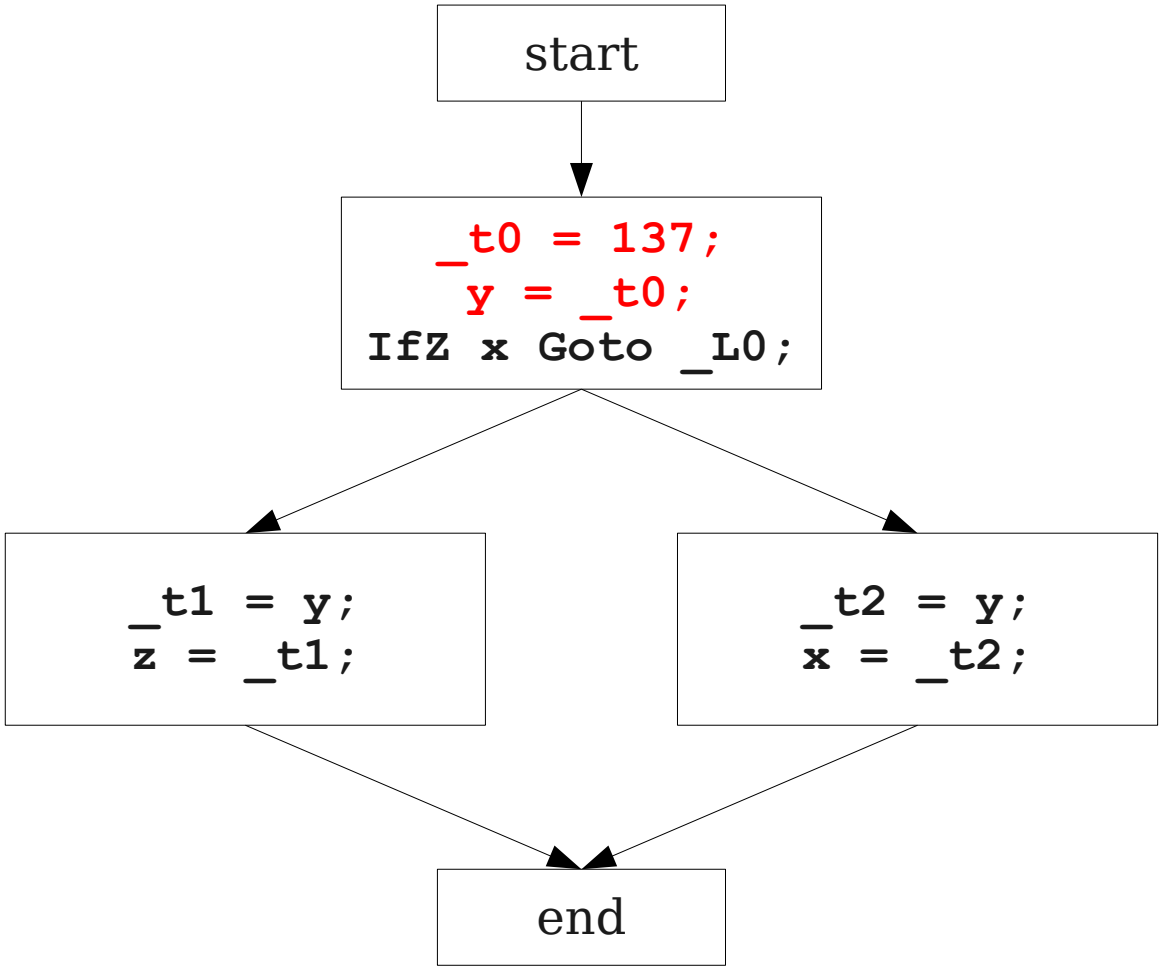

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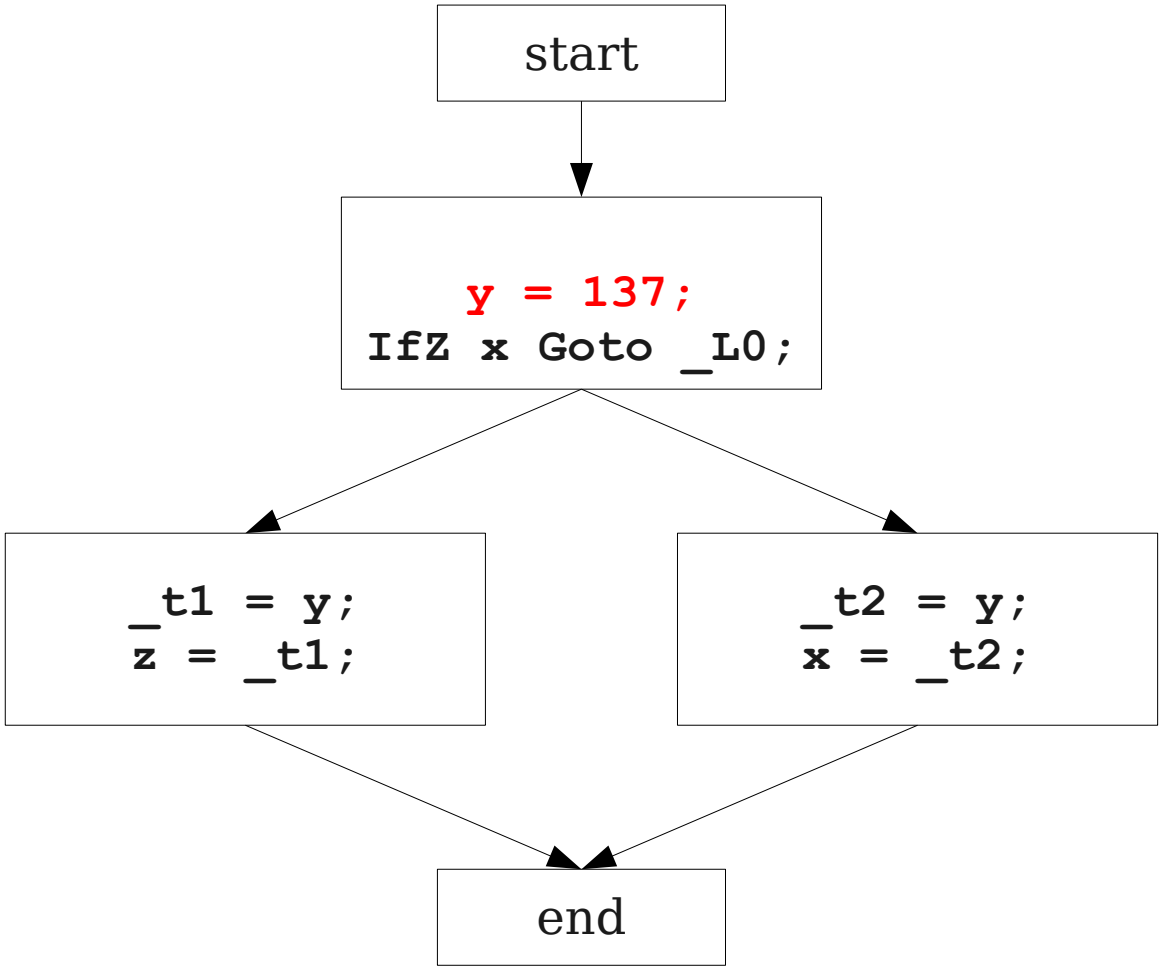

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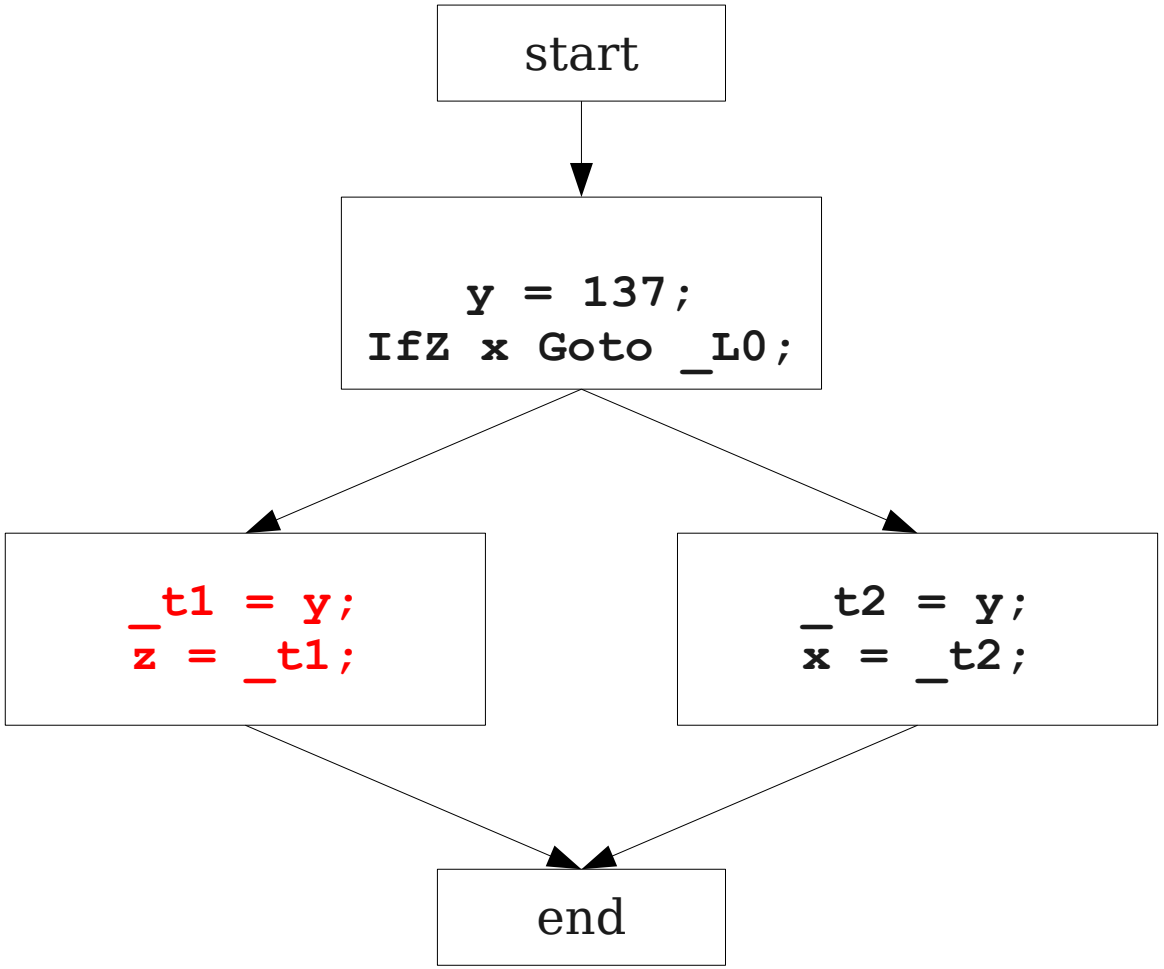

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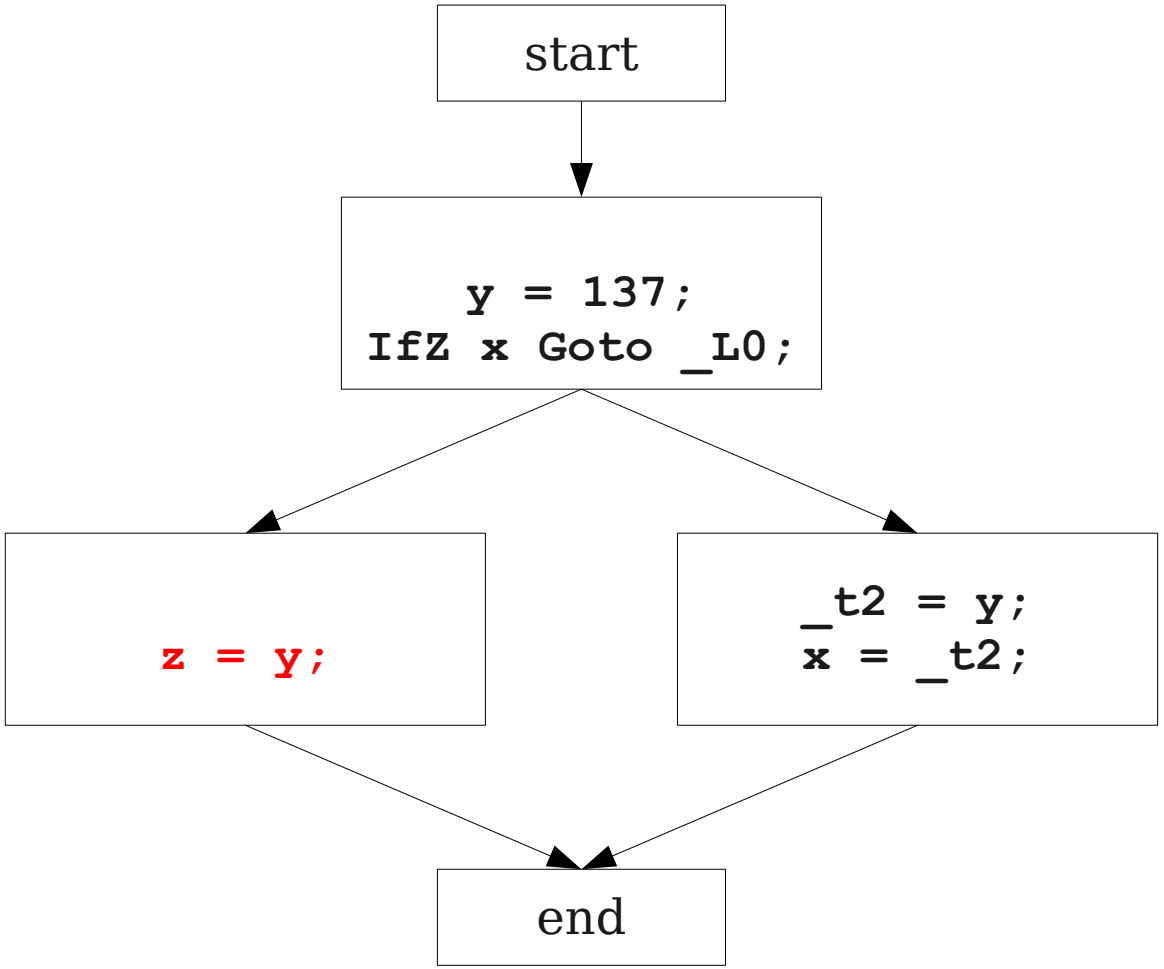

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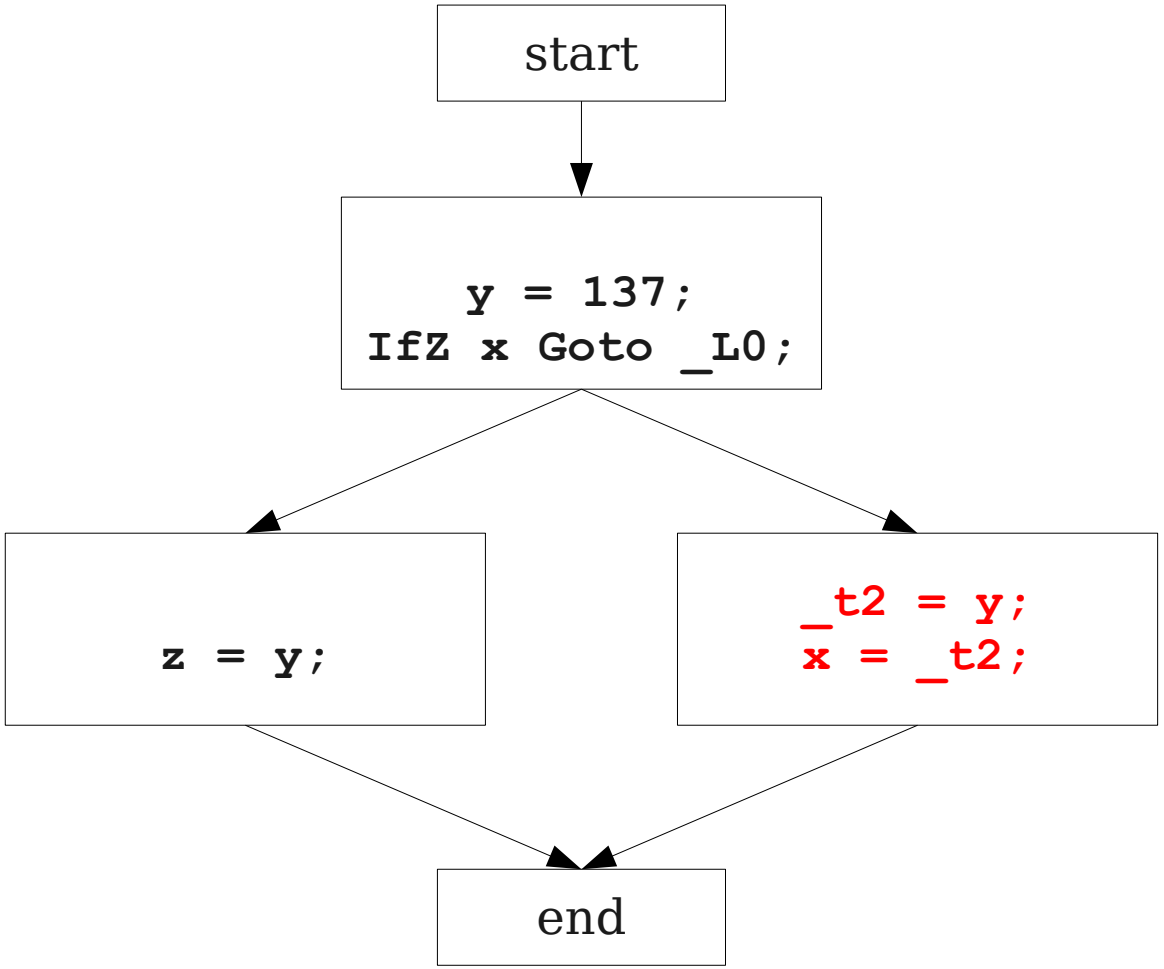

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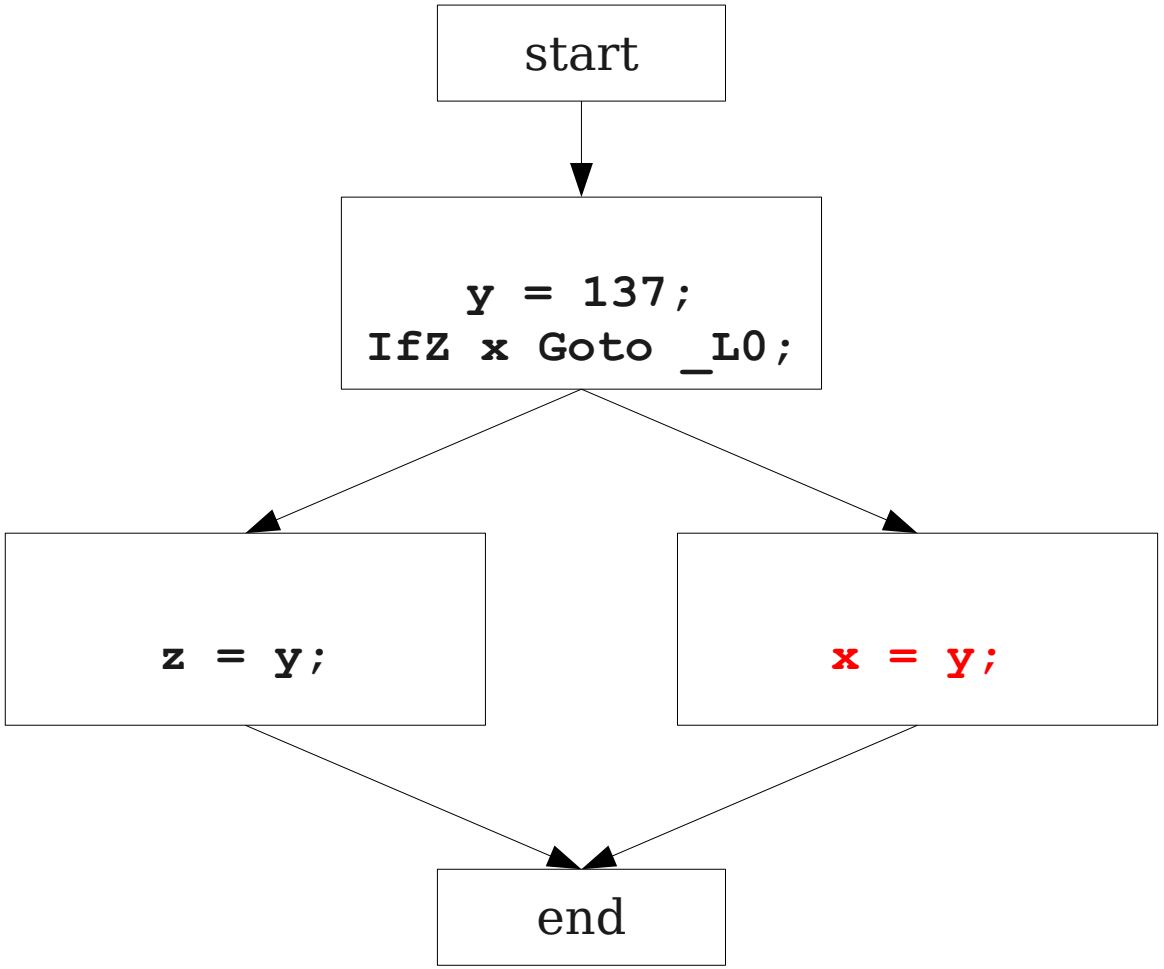

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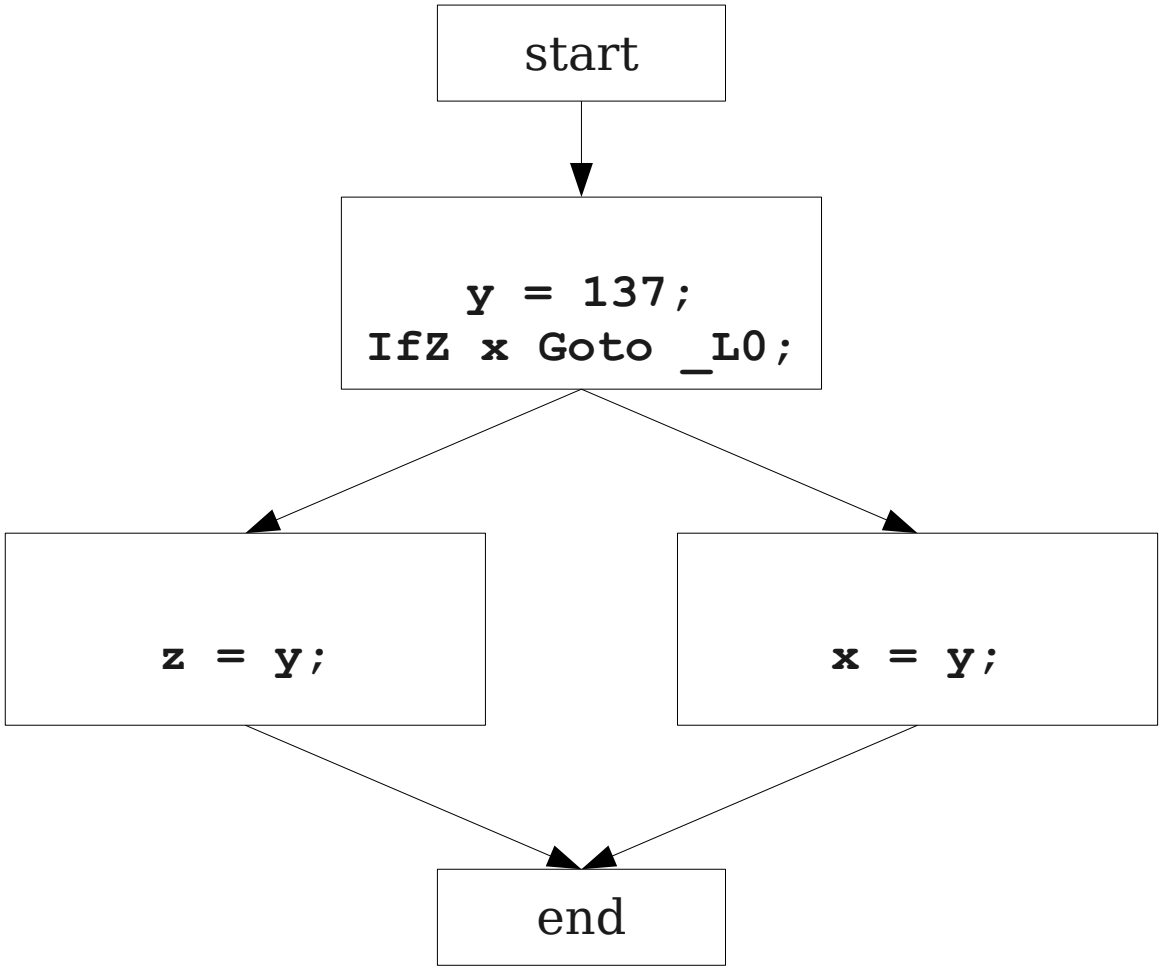

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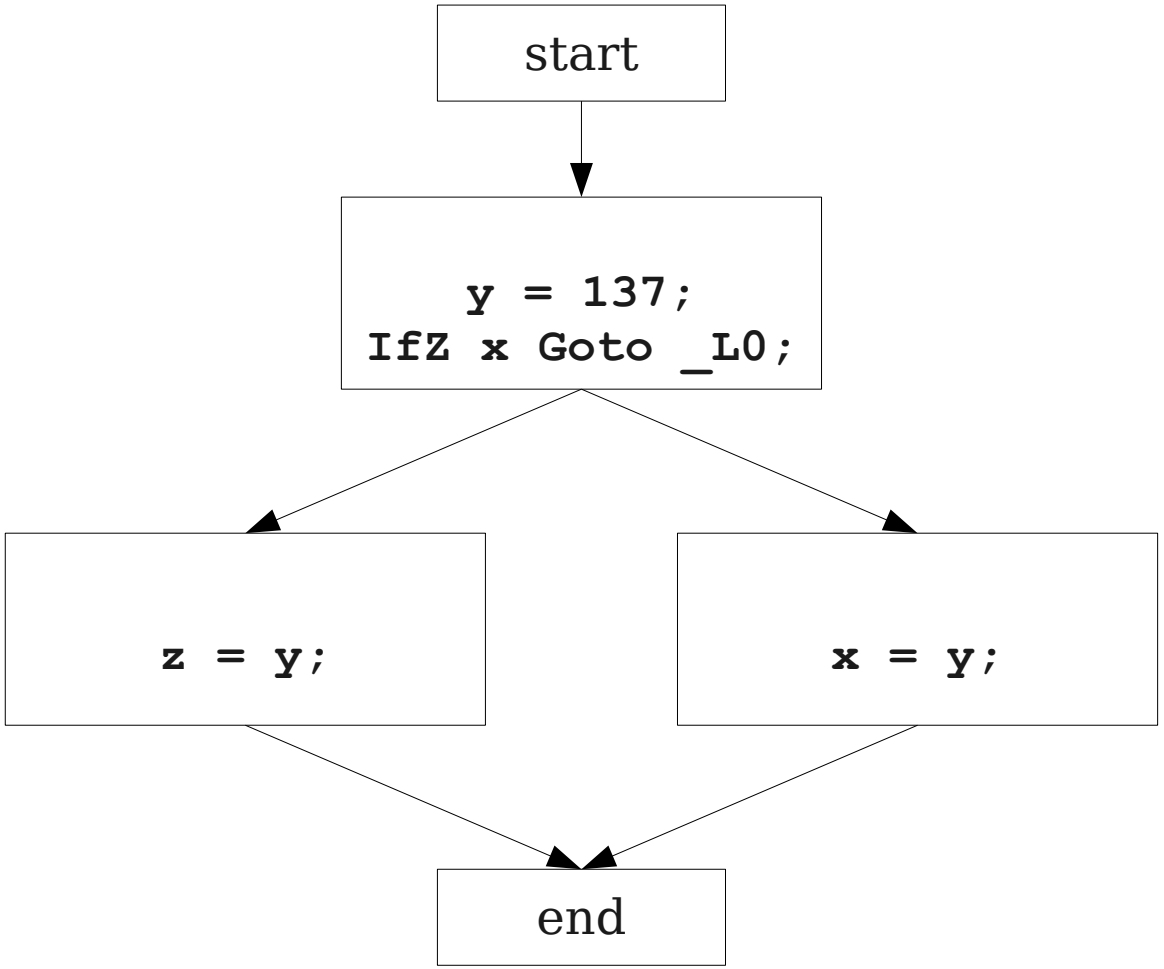

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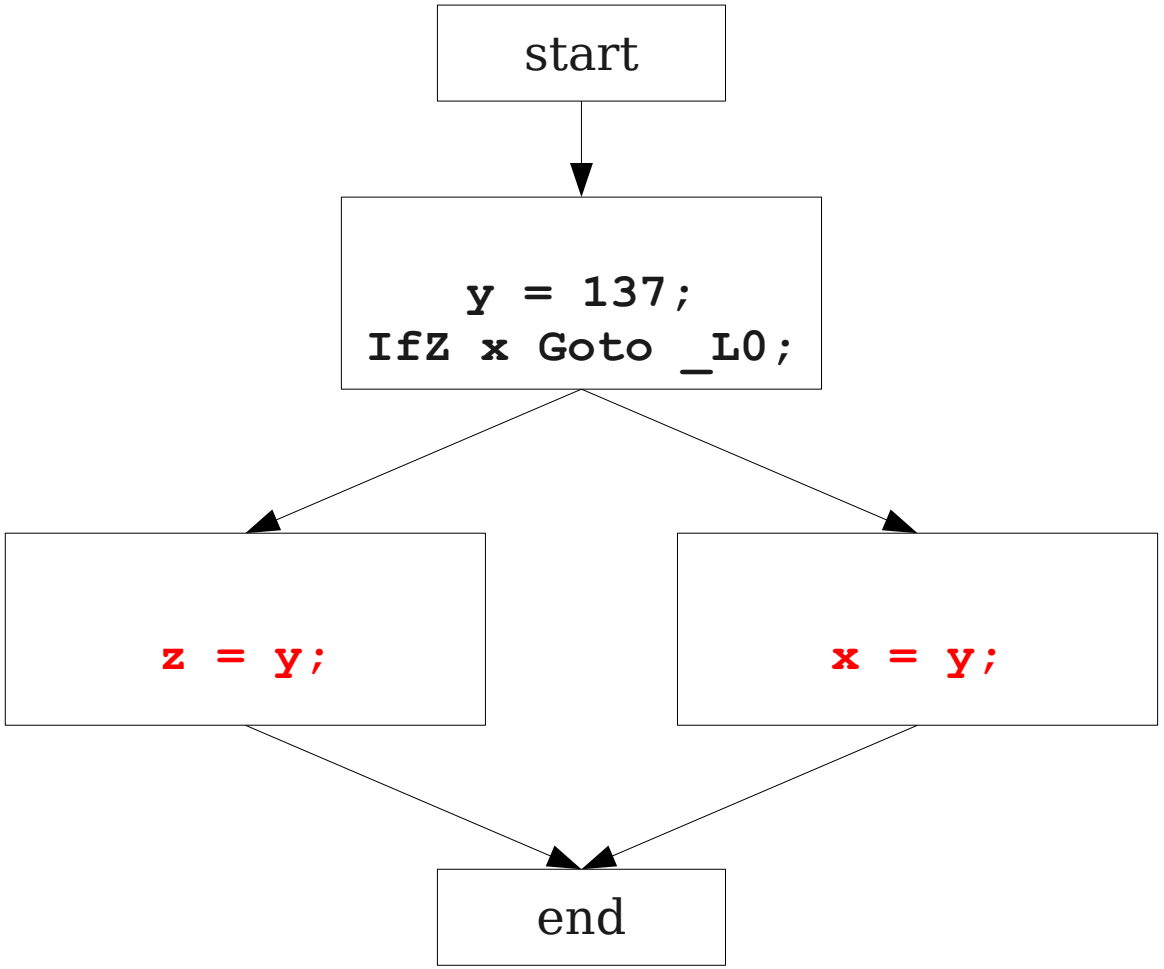

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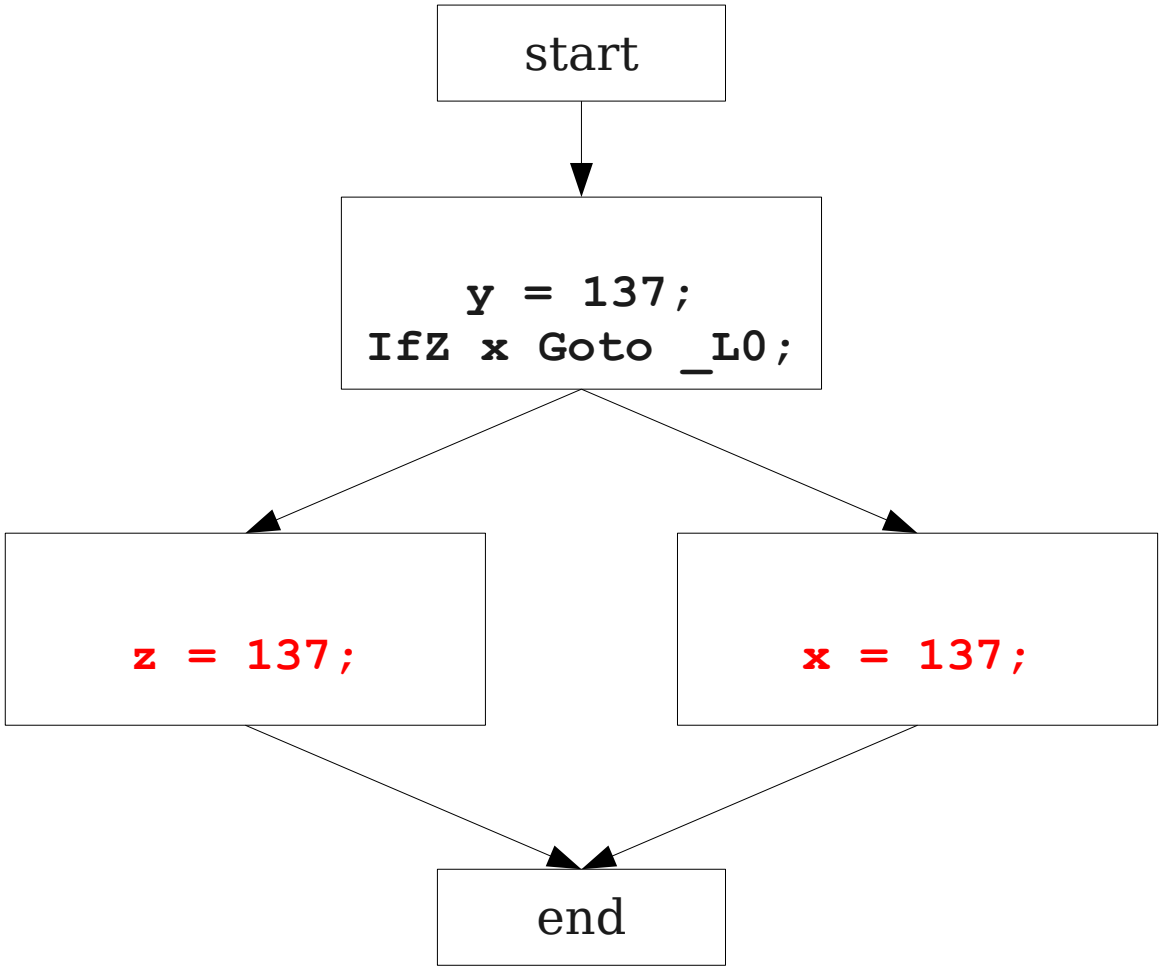

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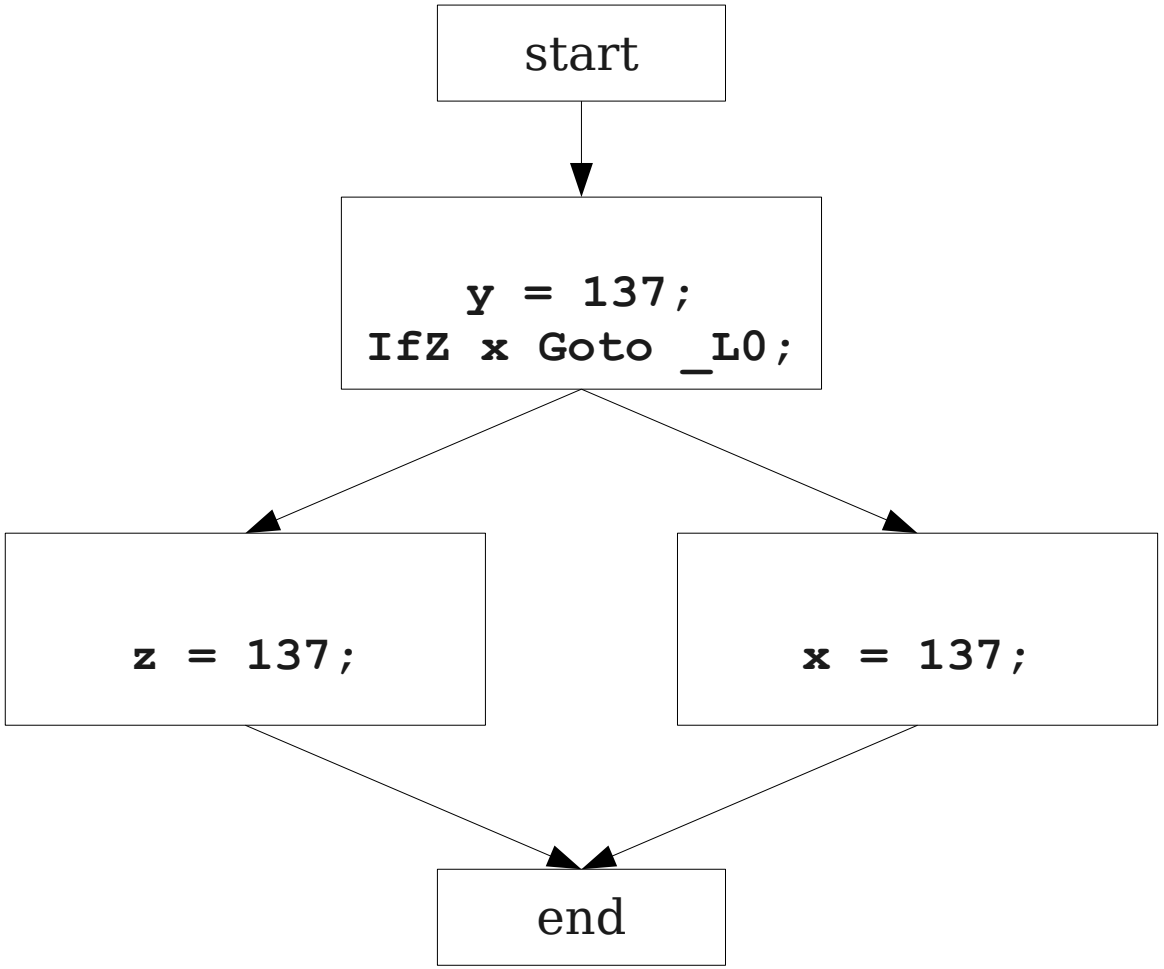
    else
        x = y;
}
```



Local Optimizations

Common Subexpression Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

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```

Common Subexpression Elimination

- If we have two variable assignments

  $\mathbf{v_1}$ `= a op b`

  ...

  $\mathbf{v_2}$ `= a op b`

  and the values of $\mathbf{v_1}$, **a**, and **b** have not changed between the assignments, rewrite the code as

  $\mathbf{v_1}$ `= a op b`

  ...

  $\mathbf{v_2}$ `=` $\mathbf{v_1}$

- Eliminates useless recalculation.
- Paves the way for later optimizations.

Copy Propagation

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```

Copy Propagation

- If we have a variable assignment

  $v_1 = v_2$

  then as long as $\mathbf{v_1}$ and $\mathbf{v_2}$ are not reassigned, we can rewrite expressions of the form

  $a = \ldots v_1 \ldots$

  as

  $a = \ldots v_2 \ldots$

  provided that such a rewrite is legal.
- This will help immensely later on, as you'll see.

Dead Code Elimination

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```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp4 = _tmp0 + b ;

_tmp5 = _tmp4 ;
_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp4 = _tmp0 + b ;

_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp4 = _tmp0 + b ;

_tmp6 = _tmp2 ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

```
Object x;
int a;
int b;
int c;

x = new Object;
a = 4;
c = a + b;
x.fn(a + b);
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;

_tmp4 = _tmp0 + b ;

_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Dead Code Elimination

- An assignment to a variable **v** is called **dead** if the value of that assignment is never read anywhere.
- **Dead code elimination** removes dead assignments from IR.
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned.

For Comparison

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
x = _tmp1 ;
_tmp3 = 4 ;
a = _tmp3 ;
_tmp4 = a + b ;
c = _tmp4 ;
_tmp5 = a + b ;
_tmp6 = *(x) ;
_tmp7 = *(_tmp6) ;
PushParam _tmp5 ;
PushParam x ;
ACall _tmp7 ;
PopParams 8 ;
```

```
_tmp0 = 4 ;
PushParam _tmp0 ;
_tmp1 = LCall _Alloc ;
PopParams 4 ;
_tmp2 = Object ;
*(_tmp1) = _tmp2 ;
_tmp4 = _tmp0 + b ;
_tmp7 = *(_tmp2) ;
PushParam _tmp4 ;
PushParam _tmp1 ;
ACall _tmp7 ;
PopParams 8 ;
```

Applying Local Optimizations

- The different optimizations we've seen so far all take care of just a small piece of the optimization.
  - Common subexpression elimination eliminates unnecessary statements.
  - Copy propagation helps identify dead code.
  - Dead code elimination removes statements that are no longer needed.
- To get maximum effect, we may have to apply these optimizations numerous times.

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = a * a;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Common Subexpression Elimination

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + c;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Copy Propagation

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = b + b;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Common Subexpression Elimination (Again)

Applying Local Optimizations

```
b = a * a;
c = b;
d = b + b;
e = d;
```

Other Types of Local Optimization

- **Arithmetic Simplification**
  - Replace “hard” operations with easier ones.
  - e.g. rewrite `x = 4 * a;` as `x = a << 2;`
- **Constant Folding**
  - Evaluate expressions at compile-time if they have a constant value.
  - e.g. rewrite **x = 4 * 5;** as **x = 20;**.

Implementing Local Optimization

Optimizations and Analyses

- Most optimizations are only possible given some analysis of the program's behavior.
- In order to implement an optimization, we will talk about the corresponding program analyses.

Available Expressions

- Both common subexpression elimination and copy propagation depend on an analysis of the **available expressions** in a program.
- An expression is called **available** if some variable in the program holds the value of that expression.

Intuition: determines for each point in the program the set of expressions that need not be recomputed. Those expressions are said to be available at such a point.

To be available on a program point, the operands of the expression should not be modified on any path from the occurrence of that expression to the program point.

Finding Available Expressions

- Initially, no expressions are available.
- Whenever we execute a statement **a = b + c**:
  - Any expression holding **a** is invalidated.
  - The expression **a = b + c** becomes available.
- **Idea**: Iterate across the basic block, beginning with the empty set of expressions and updating available expressions at each variable.

Available Expressions

```
a = b;

c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;

c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;

d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;

d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;

f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
```

Available Expressions

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = b;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = a + b;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = b;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = a + b;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
{ }
a = b;
{ a = b }
c = a;
{ a = b, c = b }
d = a + b;
{ a = b, c = b, d = a + b }
e = d;
{ a = b, c = b, d = a + b, e = a + b }
d = a;
{ a = b, c = b, d = b, e = a + b }
f = e;
{ a = b, c = b, d = b, e = a + b, f = a + b }
```

Common Subexpression Elimination

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

Live Variables

- The analysis corresponding to dead code elimination is called **liveness analysis**.
- A variable is **live** at a point in a program if later in the program its value will be read before it is written to again.
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables.

Computing Live Variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in reverse order.
- Initially, some small set of values are known to be live (which ones depends on the particular program).
- When we see the statement **a = b + c**:
  - Just before the statement, **a** is not alive, since its value is about to be overwritten.
  - Just before the statement, both **b** and **c** are alive, since we're about to read their values.
  - *(what if we have* **a = a + b***?)*

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

{ b, d }

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;
```

{ b, d, e }

```
f = e;
```

{ b, d }

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;
```

{ a, b, e }

```
d = a;
```

{ b, d, e }

```
f = e;
```

{ b, d }

Liveness Analysis

```
a = b;

c = a;

d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;
{ a, b }

{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }

{ b, d }
```

Dead Code Elimination

```
a = b;

d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;

d = a + b;

e = d;

d = a;
```

Liveness Analysis II

```
a = b;

d = a + b;

e = d;

d = a;
```

{ b, d }

Liveness Analysis II

```
a = b;

d = a + b;

e = d;
```

{ a, b }

```
d = a;
```

{ b, d }

Liveness Analysis II

```
a = b;
```

```
d = a + b;
```

{ a, b, d }

```
e = d;
```

{ a, b }

```
d = a;
```

{ b, d }

Liveness Analysis II

```
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Liveness Analysis II

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }
e = d;
{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
{ b }
a = b;

{ a, b }

d = a + b;
{ a, b, d }

{ a, b }
d = a;
{ b, d }
```

Dead Code Elimination

```
a = b;

d = a + b;

d = a;
```

Liveness Analysis III

```
a = b;

d = a + b;

d = a;
```

Liveness Analysis III

```
a = b;
```

```
d = a + b;
```

```
d = a;
```

{b, d}

Liveness Analysis III

```
a = b;
```

```
d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

Liveness Analysis III

```
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Liveness Analysis III

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}

d = a + b;

{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
{b}
a = b;

{a, b}



{a, b}

d = a;
{b, d}
```

Dead Code Elimination

```
a = b;
```

```
d = a;
```

A Combined Algorithm

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;

d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;

e = d;
{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;

c = a;

d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

d = a + b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;

c = a;

{a, b}
d = a;

{b, d}
```

A Combined Algorithm

```
a = b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

{b}

```
a = b;
```

{a, b}

```
d = a;
```

{b, d}

A Combined Algorithm

```
a = b;
```

```
d = a;
```

Next Time

- **Formalisms for Local Optimizations**
  - Transfer functions and semantics
- **Global optimization**
  - Optimizing across basic blocks.
  - Meet operators and the dataflow framework.